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Partners of the Θ^+ in large N_c QCD

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Abstract

A strangeness $+1$ exotic baryon Θ^+ has recently been seen in a number of experiments. We demonstrate that in large N_c QCD the existence of such a state implies the existence of $S = +1$ partner states with various spins and isospins but comparable masses. We discuss the spectroscopy of such states and possible channels in which they can be observed, based on the simple assumption that those states with pentaquark quantum numbers are unlikely to be large N_c artifacts.

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The recent experimental observation announced by several groups [1–4] of a strangeness $S = +1$ baryon Θ^+ with a mass 1540 MeV and narrow width (< 25 MeV) into the channel KN ranks among the most exciting findings in hadronic physics in recent years. An $S = +1$ state necessarily contains an $\bar{s}$ valence quark, whereas all previously known baryons have quantum numbers that can be accommodated by three quarks and no antiquarks. Given the KN decay channel (as distinct from $\bar{K}N$, in which conventional $S = -1$ resonances such as $\Lambda(1405)$ occur), the most natural valence content is that of a pentaquark $uudd\bar{s}$ state, an entirely new type of hadron.

The width of the Θ^+ has thus far only an experimental upper bound. While its small size may at first glance seem surprising, it is not an uncommon feature among the lowest strongly-decaying strange baryon resonances (e.g., $\Gamma[\Lambda(1520)] = 16$ MeV), and can be largely attributed to the smallness of available phase space.[1]

Exotic baryon states were studied previous to their observation, with some studies appearing as early as the late 1970s [6–8]—after all, there is no compelling reason that such states should not occur in QCD. The recent announcements by the experimental groups have spawned a flurry of theoretical work [9–11] using such tools as quark models with bags, potentials,

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[1] It has been argued, however, that the widths reported in the experiments may be broadened by experimental issues associated with resolution. Comparison with previous data seems to suggest that the actual width may be much narrower. This argument is detailed in [5].

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and pure group theory, and chiral soliton models. Nevertheless, all theoretical approaches used up to this point are more or less model dependent; in this Letter we obtain model-independent predictions, based upon the existence of the Θ^+, using only the large N_c limit of QCD.

We should note at the outset that there is no known model-independent way to predict Θ^+ properties directly from large N_c QCD. Indeed, claims that chiral soliton model predictions are independent of model details such as the profile function are shown in Ref. [12] to be the result of a treatment of collective quantization that is inconsistent with large N_c scaling.

However, large N_c analysis allows one to *correlate* predictions of exotic states. Thus, while large N_c analysis by itself cannot predict the Θ^+ mass, it *can* predict the existence of other $S = +1$ states with similar masses and widths (i.e., which differ by an amount of order $1/N_c$). The quantum numbers of such states are derived here.

In the generalization of QCD from 3 to N_c colors, the ground-state band of baryons fills a completely symmetric spin–flavor representation that subsumes the old SU(6) **56**-plet containing the N, Δ, Ω, and so on [13]. Such states have masses of $O(N_c^1)$, because N_c valence quarks are required to build a color-singlet state. Baryons within this multiplet with the same number of strange quarks are split in mass only at $O(1/N_c)$ [14]; indeed, the Δ has enough phase space for strong decays only because chiral symmetry makes the π mass smaller than the $O(1/N_c)$ Δ–N mass splitting.

Excited baryons exist as well in large N_c [15,16]. While such resonant states strictly speaking appear as poles in meson–baryon scattering amplitudes, the consistency between this picture and that of excited baryons as N_c-quark states collected into SU(6) $\times$ O(3) representations in the large N_c limit has been thoroughly demonstrated [17]. Generically, the well-known Witten N_c power counting [18] predicts excited baryons to have widths of $O(N_c^0)$ and masses above those of the ground-state band by $O(N_c^0)$. The possibility that certain baryons (those in a mixed-symmetric spin–flavor representation) are actually characteristically narrower—with widths of $O(1/N_c)$—has been discussed [16]. However, it has been recently demonstrated that this result is a consequence of the very specific simple model used; generically in large N_c QCD the excited baryon widths are, in fact, $O(N_c^0)$ [19]. With the inclusion of a typical hadronic scale Λ_{QCD}, these excitations typically amount to a few hundred MeV.

We assume, as is typically done in large N_c studies, that all quantized observables associated with baryons (i.e., spin, isospin, and strangeness) retain their $N_c = 3$ values for arbitrary N_c. Otherwise, one is faced with phenomenological consequences for $N_c > 3$ that do not match those of $N_c = 3$. In this spirit, here we assume that the appropriate large N_c generalization of the Θ^+ is a state with the quantum numbers of $(N_c + 1)$ light valence quarks and one valence strange antiquark; the additional light quarks in the large N_c world form isosinglet, spin-singlet ud pairs.

There is an important distinction between exotic meson and baryon states at large N_c. For mesons as $N_c \to \infty$ we know that there are no narrow $qq\bar{q}\bar{q}$ exotic states [20], but there must be narrow hybrid exotics with the quantum numbers of $q\bar{q}g$ [21]. For baryons, large N_c neither implies nor precludes the existence of exotic states. However, such states would still fall into nearly degenerate multiplets at large N_c in a manner analogous to the multiplet structures that arise for nonexotic baryons [17,22]. Thus, once the existence of just one such state is established, the existence of a number of others with different values of spin and isospin is guaranteed. The physics underlying this result is the existence of a small number of "reduced" scattering amplitudes, each of which contributes to a number of observable scattering amplitudes [23,24]. A complex pole appearing in one of the reduced amplitudes indicates the presence of a resonant state, with mass and width given by the real and imaginary parts, respectively. Furthermore, poles with these same values then appear in several partial waves, indicating degenerate states in the large N_c limit. In fact, since the scattering amplitudes themselves are $O(N_c^0)$, the masses and widths are degenerate to this order, and are split only at $O(1/N_c)$, with typical sizes for $N_c = 3$ of < 100 MeV.

We are interested in the appearance of such related resonant states in KN scattering amplitudes. These states have $S = +1$ and are manifestly exotic. Since strangeness plays an essential role in this problem, it may seem natural to work in an SU(3) flavor-symmetric framework. But we avoid this for a number of reasons, both practical and theoretical. On the practical side, our purpose is to predict exotic states that may be identified experimentally. $S = +1$ provides a clean experimental signature of the exotic nature of the state, and thus we focus on the $S = +1$ states. One does not require a description of full SU(3) multiplets to study these states, but only the SU(2) multiplets in the $S = +1$ subspace. On the theory side, a number of issues suggest that it is sensible to restrict ourselves to SU(2) flavor. For one, SU(3) breaking is not manifestly small for all observables—while a perturbative treatment around an SU(3)-symmetric theory works well for many observables, it also fails for some (e.g., the vector meson mass spectrum). We do not know a priori how well it can be expected to work for exotic baryons, as no one has prior experience with such states; thus, it seems prudent to refrain from relying upon SU(3) symmetry. Furthermore, there are subtleties associated with SU(3) representations at large N_c; the SU(3) representations for baryons are all infinite-dimensional as $N_c \to \infty$ [13,25]. Thus the association of representations at large N_c with representations at $N_c = 3$ is not totally trivial. In particular, if one inserts $N_c = 3$ in one part of the calculation in order to get the physical representations, one loses the ability to track the N_c [12]. While it is possible to formulate carefully the SU(3) problem at large N_c, it is less ambiguous and more physically transparent to avoid these problems by imposing only SU(2) isospin symmetry.

Consider meson–baryon scattering $m + B \to m' + B'$ with fixed strangeness in the initial and final state, such that the meson m (m') has spin s (s') and isospin i (i'). The baryon B (B') belongs to the ground-state band (N, Δ, etc.), which contains for strangeness 0 only states with spin = isospin R (R'). The total *spin* angular momentum of the meson–baryon system is denoted S (S') (and should not be confused with strangeness), while the relative orbital angular momentum is denoted L (L'). The total isospin and angular momentum of the state are denoted by I and J, respectively. Finally, abbreviate the multiplicity $2X + 1$ of an SU(2) representation of quantum number X by $[X]$. The fundamental expression relating scattering amplitudes in the large N_c limit then reads [24]

$$\begin{aligned} S_{LL'SS'IJ} &= \sum_{K,\tilde{K},\tilde{K}'} [K]\big([R][R'][S][S'][\tilde{K}][\tilde{K}']\big)^{1/2} \\ &\quad \times \begin{Bmatrix} L & i & \tilde{K} \\ S & R & s \\ J & I & K \end{Bmatrix} \begin{Bmatrix} L' & i' & \tilde{K}' \\ S' & R' & s' \\ J & I & K \end{Bmatrix} \tau_{K\tilde{K}\tilde{K}'LL'}. \end{aligned} \tag{1}$$

A few words about the derivation of Eq. (1) are in order. This equation was originally derived in the context of an SU(2) Skyrme-type model (i.e., a model without a strange degree of freedom), and with the help of standard identities in SU(2) group theory was then used to deduce the $I_t = J_t$ rule for meson–baryon scattering [24]. As noted in Ref. [17], such a derivation can be turned on its head: the fact that large N_c consistency rules can be shown to imply the $I_t = J_t$ rule for all observables at large N_c [26], together with the same SU(2) identities, implies that Eq. (1) holds at leading order in $1/N_c$. Such a derivation is fully model independent. Moreover, this alternative derivation makes clear that Eq. (1) applies to the scattering of strange mesons off nonstrange baryons. The point is simply that the derivation depends only on the quantum numbers exchanged in the t channel; these quantum numbers are necessarily nonstrange for such a scattering since the strange quark both enters and leaves the reaction in the meson, without being transferred to the baryon. On the other hand, resonances in the s channel then have $S = +1$, allowing one to make contact with the exotic states of interest.

In the present case the mesons are kaons and thus $s = s' = 0$, which collapses the $9j$ symbols to $6j$ symbols, forcing as well $S \to R$, $S' \to R'$, and $\tilde{K} = \tilde{K}' = K$. The reduced amplitudes may then be relabeled $s^{\mathcal{K}}_{KLL'} = (-1)^{L-L'}\tau_{KKKLL'}$ ($\mathcal{K}$ denotes the kaon). Furthermore, $i = i' = 1/2$. In this case the expression simplifies to

$$\begin{aligned} S_{LL'RR'IJ} &= [R]^{1/2}[R']^{1/2}(-1)^{R-R'} \\ &\quad \times \sum_K [K] \begin{Bmatrix} \frac{1}{2} & L & K \\ J & I & R \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & L' & K \\ J & I & R' \end{Bmatrix} s^{\mathcal{K}}_{KLL'}. \end{aligned} \tag{2}$$

All of the predictions of this Letter follow simply from this relation.

Each distinct pole occurring in Eq. (2) is identified solely by the K value of the reduced amplitude; the L, L' values refer to the manner from which meson–baryon scattering states couple to the resonances but do not characterize the resonance itself.

For the purposes of this Letter, we assume that the observed Θ^+ is an $I=0$ state. At present there is no definitive experimental evidence for this value. However, it does emerge as the lowest state in many models. In any case, we will take this as a starting point for the analysis here both because it is quite plausible and because the analysis is particularly simple. If it turns out subsequently that the Θ^+ has a different isospin, the analysis can be easily modified to account for the correct value.

The isospin of the Θ^+ is not the only unknown quantum number; its spin and parity also have not been fixed experimentally. Our prediction of partner states with given quantum numbers depends upon these values for the Θ^+. Once they are fixed from experiment, one can make a concrete predictions. In the following we make predictions for partner states based on all of the possible values $J_0^{P_0}$ consistent with an $N_c=3$ pentaquark. Note that Eq. (2) with $I=0$ is exceptionally simple: the only surviving amplitudes are

$$S_{L_0 L_0 \frac{1}{2} \frac{1}{2} 0 J_0} = s^{\mathcal{K}}_{J_0 L_0 L_0}. \tag{3}$$

In particular, the only allowed K value equals J_0, only $R=R'=\frac{1}{2}$ (the pole appears in KN, but not $K\Delta$, channels due to I conservation), and $L_0' = L_0$ (the pole does not appear in mixed partial waves due to angular momentum and parity conservation). From here, the procedure is extremely straightforward: one looks for channels with distinct (I,J) values in which poles with $K=J_0$ occur.

The results obtained from Eq. (2) are straightforward to summarize. A state of $I=0$ and spin J_0 and either parity P_0 occurring in a partial wave of relative orbital angular momentum L_0 has $I=1$ partners (meaning masses and widths equal to within $O(1/N_c)$) of all spins J_1 consistent with the vector addition $\mathbf{J_1} = \mathbf{J_0} + \mathbf{1}$, and the same parity $P_1 = P_0 \equiv P$. Furthermore, the $I=1$ partners appear only in partial waves with the same orbital quantum number, $L_1 = L_0 \equiv L$, which is constrained by angular momentum conservation to lie within $1/2$ unit of J_0. These latter two results rely on parity conservation.

The previous scheme lists all the possible partner states for a world with $N_c \to \infty$. Clearly some of these could be large N_c artifacts. On the other hand, the J_0 values that may be constructed as true pentaquarks at $N_c=3$, namely, $1/2$, $3/2$, and $5/2$, are physically very plausible. Moreover, for any $I=0$ state containing a (necessarily equal) number of u and d quarks, it is possible to construct one of the same quark content—and hence for the same value of N_c—but $I=1$ by flipping relative signs of the u–d flavor wave function. One then finds, for either value of P, that $I=1$ partners with each allowed J_1 appear in all channels coupled to either KN $(R=1/2)$ or $K\Delta$ $(R=3/2)$, with the following exceptions:

- If $J_0^P = 1/2^-$, then only amplitudes with $J_1 = R = R'$ contain the pole with $K=J_0$ (i.e., $KN \to K\Delta$ does not produce this resonance).
- If either (1) $J_0^P = 3/2^+$ and $J_1^P = 5/2^+$, (2) $J_0^P = 3/2^-$ and $J_1^P = 1/2^-$, (3) $J_0^P = 5/2^-$ and $J_1^P = 7/2^-$, or (4) $J_0^P = 7/2^+$ and $J_1^P = 3/2^+$, then the $K=J_0$ pole appears only in amplitudes with $R = R' = 3/2$. In such cases, the given partners would not be visible in $KN \to KN$ or $KN \to K\Delta$ processes, and therefore alternate experiments to KN scattering would be required to uncover the existence of such partners.

If any of the pentaquark states produced in KN scattering—whose total spin angular momentum is only $1/2$—should have a large spin such as $5/2$ or $7/2$, then it must be produced in a high partial wave, say $L=2$ or 3. Assuming that the mass of the state is near the KN threshold, as is true for the Θ^+ and therefore also for its partners (degenerate to within about 100 MeV), then the available phase space is proportional to $|\mathbf{p}|^{2L+1}$ and quite small widths ($O(1$ MeV$)$ or less) for such states are not out of the question.

Eq. (2) also predicts $I=2$ partners to the $I=0$, spin J_0 state in large N_c. One can show that for either parity, partners with each spin allowed by the vector addition rule $\mathbf{J_2} = \mathbf{J_0} + \mathbf{2}$ occur; in the context of KR scattering, where again R is a ground-state band nonstrange baryon with $I=J$, $I=2$ states may only be reached through the $K\Delta$, and some require scattering with the $N_c \geqslant 5$ baryon with $R = 5/2$. Nevertheless, $I = 2$ states may be reached through other channels such γd. An important caveat to keep in mind when considering higher-isospin partners, however, is that such states may be artifacts of $N_c > 3$, since their total isospin may include u, d valence quarks beyond the four available in the $N_c = 3$ pentaquark.

Thus we see, regardless of the spin and parity of the Θ^+, that large N_c QCD predicts it has quantum-number exotic partners degenerate in mass and width up to $O(1/N_c)$ effects. While one does expect the mass prediction to work fairly well—states within a couple of hundred MeV—a note of caution should be added about the widths. The Θ^+ is rather close to threshold, far closer than is "natural" from $1/N_c$ effects alone. As a result, phase space greatly restricts its width. In contrast, its partner states may be expected to be significantly higher above threshold, which greatly increases the phase space. Accordingly, one does not necessarily expect the widths of these partner states to be similar to that of the Θ^+, but similar to each other.

We note lastly that Eq. (2) holds for any fixed value of S (the strange quarks merely "go along for the ride"), as long as the mesons in the scattering process have $i = 1/2$. That is, Eq. (2) works equally well for $\bar{K}N$ scattering. Our results predicting the partners of the Θ^+ carry over verbatim to predictions of partners of the Λ resonances. They imply that a Λ with spin–parity J_0^P appearing in a partial wave of a given L has Σ partners with spins satisfying the vector addition rule $\mathbf{J_1} = \mathbf{J_0} + \mathbf{1}$, in the same partial wave L, with the same parity P. This analysis does *not*, however, predict the multiplicity of states with degenerate I, J^P corresponding to the same K pole but distinguished by quantum numbers not specified here. An excellent example is provided by the $\Lambda(1405)$ and the $\Lambda(1670)$, both of which have $I = 0$ and $J^P = 1/2^-$, and both are generally assigned to the mixed-symmetry spin–flavor multiplet of SU(6) (a **70** for $N_c = 3$), but the former is an SU(3) singlet and the latter is in an SU(3) octet for $N_c = 3$. Eq. (3) indicates that both states correspond to $K = 1/2$ poles, but because the masses can be accommodated by (substantial but not particularly large) $O(1/N_c)$ corrections [15], while distinct poles with a given K are generically expected to be separated by $O(N_c^0)$, one concludes that they both originate from the *same* $K = 1/2$ pole.

One then concludes that in large N_c, regardless of any particular picture such as the quark model, Λ resonances of a given spin–parity should always have Σ partners with quantum numbers as described above. This is a testable proposition, for which a quick survey of the *Review of Particle Physics* [27] is appropriate. One finds that the $J_0^P = 1/2^-$ S_{01} states $\Lambda(1405)$ and $\Lambda(1670)$ appear to have as a partner the $J^P = 1/2^-$ S_{11} $\Sigma(1620)$, while the $J_0^P = 1/2^-$ S_{01} state $\Lambda(1800)$ appears to be partnered with the $J^P = 1/2^-$ S_{11} $\Sigma(1750)$. The $3/2^-$ D_{03} resonances $\Lambda(1520)$ and $\Lambda(1690)$ should be partnered with $3/2^-$ and $5/2^-$ Σ's visible in $\bar{K}N$ scattering, and indeed there appear to exist D_{13} $3/2^-$ states $\Sigma(1580)$, $\Sigma(1670)$, and $\Sigma(1940)$, and the D_{15} state $\Sigma(1775)$. One more example is appropriate: the $1/2^+$ P_{01} states $\Lambda(1600)$ and $\Lambda(1810)$ should be partnered with $1/2^+$ and $3/2^+$ Σ's and indeed there exist P_{11} $\Sigma(1660)$ and $\Sigma(1880)$, but the evidence for the P_{13} states below 2 GeV is still poor. While a number of predicted Σ states have not yet been seen definitively, each one that has been observed can be identified as being partnered with some observed Λ state.

In summary, the large N_c limit of QCD provides a powerful tool to determine multiplets of baryon states related by symmetry, even in interesting cases like that of the Θ^+, where the detailed dynamics underlying their mere existence remains obscure.

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